Answer ALL the following questions.

1. When each of the polynomials \( f(x) = 2x^2 + 3x - 1 \) and \( g(x) = A(x+1)(x+2) + B(x+1)^2 + C(x+2)^2 \) is divided by \( x, \ x+1 \) and \( x+2 \) successively, the same remainder is obtained.
   (a) Find the values of \( A, B \) and \( C \).
   (b) (i) Show that \( f(x) \equiv g(x) \).
       (i) Hence solve the equation \( g(x) = 4 \).

2. (a) Find the values of the constants \( A, B \) and \( C \) in the identity
    \[ x(Ax - 3)^2 + B(2x + C) = 4x^3 - 12x^2 + (8 + C)x + 6 \]
   (b) Hence factorize \( 4x^3 - 12x^2 + 5x + 6 \) completely.
   (c) Using the result, simplify
    \[ \frac{6 - 23x}{x-2} \frac{2x^2}{(2x+1)} \frac{14x}{x-2} \]
   (d) Hence, solve the equation
    \[ \frac{6 - 23x}{(x-2)(2x+1)} = \frac{14x}{2x+1} \frac{2x^2}{x-2} \]

3. Given two polynomials \( x^3 + 5x^2 - 17x - 21 \) and \( 3x^3 + 14x^2 - 55x - 42 \),
   (a) Show that \( x - 3 \) is a common factor of the two polynomials,
   (b) Factorize the polynomials completely,
   (c) Find the H.C.F. and the L.C.M. of the two polynomials,
   (d) Show that the product of the polynomials is equal the product of their H.C.F. and L.C.M..

4. Let \( f(x) = x^{1999} + x + 1 \).
   (a) Find the remainder when
       (i) \( f(x) \) is divided by \( x+1 \)
       (ii) \( f(x) \) is divided by \( x-1 \)
   (b) Hence, find the values of \( a \) and \( b \) if \( f(x) = (x^2 - 1)g(x) + ax + b \) where \( g(x) \) is a polynomial in \( x \) and \( a, b \) are constants.
   (c) Using the result of (b), find the remainder when
       (i) \( 11^{1999} \) is divided by 120.
       (ii) \( 11^{2000} \) is divided by 120.
1. (a) put \( x = -1 \)
\[
g(-1) = f(-1)
\]
\[
\therefore C = -2
\]
put \( x = -2 \)
\[
g(-2) = f(-2)
\]
\[
\therefore B = 1
\]
put \( x = 0 \)
\[
g(0) = f(0)
\]
\[
\therefore 2A + 1 - 8 = -1
\]
\[
\therefore A = 3
\]
(b) (i) \[
g(x) = 3(x+1)(x+2)+(x+1)^2 - 2(x+2)^2
\]
\[
= 2x^2 + 3x - 1
\]
\[
= f(x)
\]
(ii) \[
g(x) = 4
\]
\[
f(x) = 4
\]
\[
2x^2 + 3x - 5 = 0
\]
\[
(2x+5)(x-1) = 0
\]
\[
\therefore x = -\frac{5}{2} \text{ or } 1
\]
2. (a) LHS= \( x(Ax - 3)^2 + B(2x + C) \)
\[
= A^2 x^3 - 6Ax^2 + (9 + 2B)x + BC
\]
By comparing the coefficient,
\[
\begin{align*}
A &= 2 \\
9 + 2B &= 8 + C \\
BC &= 6
\end{align*}
\]
\[
\therefore B = \frac{3}{2} \quad C = 4
\]
\[
\therefore B = -2 \quad C = -3
\]
(b) When \( A = 2, \ B = -2 \) and \( C = -3 \)
\[
4x^3 - 12x^2 + 5x + 6 \equiv x(2x - 3)^2 - 2(2x - 3)
\]
\[
\equiv (2x - 3)(2x^2 - 3x - 2)
\]
\[
\equiv (2x - 3)(2x + 1)(x - 2)
\]
(c) \[
\frac{6 - 23x}{(x-2)(2x+1)} + \frac{2x^2 - 14x}{x-2} \cdot \frac{2x+1}{x-2} - \frac{14x}{2x+1}
\]
\[
= \frac{6 - 23x + 2x^2(2x+1) - 14x(x-2)}{(x-2)(2x+1)}
\]
\[
= \frac{4x^3 - 12x^2 + 5x + 6}{(x-2)(2x+1)}
\]
\[
= 2x - 3
\]
(d) Since \( x \neq 2 \) or \( -\frac{1}{2} \)

By the result of (d), \( x = \frac{3}{2} \)
3. Let \( f(x) = x^3 + 5x^2 - 17x - 21 \)
\[
g(x) = 3x^3 + 14x^2 - 55x - 42
\]
(a) \[
f(3) = 3^3 + 5(3^2) - 17(3) - 21 = 0
\]
\[
g(3) = 3(3^3) + 14(3^2) - 55(3) - 42 = 0
\]
\[
\therefore (x-3) \text{ is the factor of } f(x) \text{ and } g(x).
\]
(b) \[
f(x) = (x-3)(x^2 + 8x + 7)
\]
\[
g(x) = (x-3)(3x+2)(x+7)
\]
\[
\therefore \text{ H.C.F. } = (x-3)(x+7)
\]
\[
\text{L.C.M. } = (x-3)(x+7)(3x+2)(x+1)
\]
(e) product of H.C.F. and L.C.M. is
\[
(x-3)^2(x+7)^2(3x+2)(x+1)
\]
that is equal to \( f(x) \cdot g(x) \)

4. (a) (i) \( f(-1) = -1 \)
(ii) \( f(1) = 3 \)
(b) \( f(-1) = -a + b = -1 \)
\[
f(1) = a + b = 3
\]
\[
\therefore a = 2, \ b = 1
\]
\[
\therefore f(x) = (x^2 - 1)g(x) + 2x + 1
\]
© (i) \( : x^{1999} + x + 1 = (x^2 - 1)g(x) + 2x + 1 \)
\[
x^{1999} = (x^2 - 1)g(x) + x
\]
put \( x = 11 \)
the remainder is 11.
\[
x^{2000} = (x^2 - 1)g(x+1) + 1
\]
put \( x = 11 \), the remainder is 1.