Answer ALL questions and write your answers in the spaces provided.

Unless otherwise specified, all working must be clearly shown.

Unless otherwise specified, numerical answers should either be exact or correct to 3 decimal places.

Total mark is 48 marks.

1. For each of the following numbers:
   \[ 5, 7, 4\pi, \frac{10}{3}, \sqrt{81}, 0.3, \frac{22}{7}, \sqrt{2}, -\sqrt{5} \]
   state whether it is
   (a) An irrational number, \(4\pi, \sqrt{2}\) and \(-\sqrt{5}\) .................................................. (3 marks) (3A)
   (b) A rational number but not an integer, \(10\frac{1}{3}, 0.3\) and \(\frac{22}{7}\) ............................................. (3 marks) (3A)
   (c) Natural number. .......................................................... (2 marks) (2A)

2. If \(a\) and \(b\) are real numbers,
   (a) Factorize \(a^2 - 4a + 4\) .......................................................... (1 marks) (1A)
       \[ a^2 - 4a + 4 = (a - 2)^2 \]
   (b) Hence, factorize \(a^2 - b^2 - 4a - 2b + 3\). .................................................. (4 marks) (1A)
       \[ a^2 - 4a + 4 - b^2 - 2b - 1 \]
       \[ = (a - 2)^2 - (b + 1)^2 \] .................................................. (1M+1A) (1A)
       \[ = (a + b - 1)(a - b - 3) \] .................................................. (1A)
3. Let \( f(x) = ax^2 + x + 5 \) and \( g(x) = 2x^2 + bx + 5 \). If \( f(1) = g(1) \) and \( f(0) = g(-1) \), find

(a) The values of \( a \) and \( b \),

\[
\begin{align*}
\therefore f(0) &= g(-1) \\
5 &= 2 - b + 5 \\
\therefore b &= 2 \\
\therefore f(1) &= g(1) \\
\therefore a + 6 &= 7 + 2 \\
\therefore a &= 3
\end{align*}
\]

(b) The value of \( n \) such that \( 3g(n) - 2f(n) = 1 \).

\[
3(2n^2 + 2n + 5) - 2(3n^2 + n + 5) = 1
\]

\[
4n = -4 \\
\therefore n = -1
\]

4. Given that the equation \( x^2 - x - 1 = 0 \).

(a) Solve the above equation and leave the answers in surd form.

\[
x = \frac{1 \pm \sqrt{1 + 4}}{2}
\]

\[
\therefore x = \frac{1 \pm \sqrt{5}}{2}
\]

(b) Hence, find the value of \( x^3 - x^2 + x + 1 \) for the negative value of \( x \).(Leave answer in surd form)

\[
\therefore x^2 - x - 1 = 0 \\
\therefore x^3 - x^2 - x = 0 \\
x^3 - x^2 = x \\
x^3 - x^2 + x + 1 = 2x + 1
\]

\[
= -\sqrt{5}
\]
5. If the equation \( x[(2k - 1)x + 3](3k + 1) + 9 = 0 \) has equal roots, find

(a) The value(s) of \( k \), \((5 \text{ marks})\)

\[
(2k - 1)(3k + 1)x^2 + 3(3k + 1)x + 9 = 0
\]

Since the equation has equal roots,

\[
\Delta = 0
\]

i.e. \( 9(3k + 1)^2 - 4(2k - 1)(3k + 1)(9) = 0 \) \((1M+1A)\)

\[
(3k + 1)(-5k + 5) = 0
\]

\[
\therefore k = -\frac{1}{3} \text{ or } 1
\] \((1A+1A)\)

(b) The roots of the equation for positive value of \( k \). \((2 \text{ marks})\)

The roots \( = \frac{-3(3+1)}{(2)(4)} = -\frac{3}{2} \) \((1M+1A)\)

6. If the price of an orange rises by $1, then 5 fewer oranges could be bought for $100.

(a) Find an equation in terms of \( x \) if the original price of an orange is $\( x \). \((3 \text{ marks})\)

The number of orange could be bought originally is \( \frac{100}{x} \). \((1A)\)

The number of orange could be bought after rising the price of an orange is \( \frac{100}{x+1} \). \((1A)\)

\[
\therefore \frac{100}{x} - \frac{100}{x+1} = 5
\] \((1A)\)

(b) Hence, find the value of \( x \). \((4 \text{ marks})\)

\[
x(x+1) = 20(x + 1 - x)
\] \((1M)\)

\[
x^2 + x - 20 = 0
\] \((1A)\)

\[
(x-5)(x+4) = 0
\] \((1A)\)

\[
\therefore x = 4 \quad (\because x > 0)
\] \((1A)\)
7. The figure shows the graph of \( y = ax^2 + bx + c \).

(a) Write down the value of \( c \).  

The value of \( c \) is 3  

(b) Determine whether \( a > 0 \) or \( a < 0 \).  

Since the graph is opening upward, \( a > 0 \)  

(c) Find the values of \( a \) and \( b \).  

Since the roots are \( \frac{1}{2} \) and \( \frac{3}{2} \).  

The equation should be \( y = k(2x - 1)(2x - 3) \)  

By (a), \( k = 1 \)  

\[ \therefore y = 4x^2 - 8x + 3 \]  

\[ \therefore a = 4; \ b = -8 \]