Answer ALL the following questions.

1. (a) In a geometric sequence with $A$ as the first term. The common ratio $R$ is such that $-1 < R < 1$ so that it can be summed to infinity. Let $S$ be the sum to infinity. Consider another sequence formed by the squares of the terms of the original geometric sequence.

   (i) Is this new sequence a geometric sequence?
   (ii) Can this new sequence be summed to infinity? Why?

   (iii) Show that the sum to infinity of this new sequence is $\frac{AS}{1+R}$.

   (b) If the first term and the sum to infinity of a geometric sequence are 50 and 75 respectively, 

   (i) Find the common ratio,
   (ii) Using (a), find the sum to infinity of the squares of the terms of this geometric sequence.

2. In the figure, each square is inscribed in a circle and each circle (except the first one) is drawn touching the sides of a square. This pattern continues indefinitely. Given that the radius of the greatest circle, $OA_1$, is 4 cm. Leave the answers in surd form.

   (a) Find the radii $OA_2$ and $OA_3$.
   (b) Let $P_n$ denote the circumference of the circle with radius $OA_n$

      (i) Show that $P_1, P_2, P_3, \ldots$ form a geometric sequence.
      (ii) Find the sum of the circumferences of all these circles.
   (c) Find the sum of the areas of all the squares inscribed.

3. The third term and fifth term of an arithmetic sequence are 19 and 13 respectively.

   (a) Find the first term and the common difference.
   (b) Find the first negative term of the sequence.
   (c) Let $S_n$ be the sum of the first $n$ terms. Find the largest value of $S_n$. 
1. Let $T(n)$ and $N(n)$ be general term of the original sequence and the new sequence respectively.

(a) \( T(n) = AR^{n-1} \) and \( N(n) = A^2 R^{2n-2} \)

(i) Consider \( \frac{A^2 R^{2n-2}}{A^2 R^{2(n-1)-2}} = R^2 = \text{constant} \)

Therefore, \( N(n) \) is a geometric sequence.

(ii) Since \( 0 < R^2 < 1 \), the new sequence can be summed up to infinity.

(iii) Since \( S = \frac{A}{1-R} \) and the sum to infinity of the new sequence is

\[
\frac{A^2}{1-R^2} = \frac{AS(1-R)}{(1-R)(1+R)} = \frac{AS}{1+R}.
\]

(b)

(i) \( 75 = \frac{50}{1-R} \) therefore \( R = \frac{1}{3} \)

(ii) the required sum \( = \frac{50(75)}{1+\frac{1}{3}} = \frac{5625}{2} \).

2. (a) \( A_1B_1 = \sqrt{OA_1^2 + OB_1^2} = 4\sqrt{2} \text{ cm} \) Since \( OA_2 = \frac{1}{2} A_1B_1 = 2\sqrt{2} \text{ cm} \)

\( A_2B_2 = \sqrt{OA_2^2 + OB_2^2} = 4 \) cm Since \( OA_3 = \frac{1}{2} A_2B_2 = 2 \text{ cm} \)

(c) From (a), we have \( OA_n = \left( \frac{\sqrt{2}}{2} \right)^{n-1} (4) \text{ cm}, \quad P_n = 2\pi OA_n = 8\pi \left( \frac{\sqrt{2}}{2} \right)^{n-1} \text{ cm} \)

Therefore, \( P_n \) formed a geometric sequence.

The sum of the circumference of all circles \( = \frac{8\pi}{1-\frac{\sqrt{2}}{2}} = 8\pi \left( 2 + \sqrt{2} \right) \text{ cm} \)

The sum of the area of all squares \( = \frac{(4\sqrt{2})^2}{1-\frac{1}{2}} = 64 \text{ cm}^2 \)

3. Let \( T(n) \) be the general term of the sequence.

\( T(n) = a + (n-1)d \) where \( a \) be the first term and \( d \) be the common difference., we have

\[
\begin{align*}
\begin{cases}
a + 2d = 19 \\
a + 4d = 13 \\
\end{cases}
\Rightarrow d = -3 \quad a = 25
\end{align*}
\]

i.e. \( T(n) = 28 - 3n \) when \( T(n) < 0, \quad n > \frac{28}{3} \). Since \( n \) is a positive integer, \( n = 10 \).

\( T(10) = -2 \) the greatest \( S = \frac{(25 + 1)(9)}{2} = 117 \)