Summary of Geometrical Proof Reasoning

Angles and Parallel Lines

(a) If $AOB$ is a straight line, then $a + b = 180^\circ$. (adj. $\angle$s on st. line)
(b) If $AO, FO, GO, BO$ and $EO$ meet at a point $O$, then $a + b + f + g + h = 360^\circ$ ( $\angle$s at a pt.)
(c) If $CD$ and $EG$ intersect at a point, then $c = d$ (vert. opp. $\angle$s)
(d) If $AB \parallel CD$, then
   i. $h = d$ (corr. $\angle$s, $AB \parallel CD$)
   ii. $a = d$ (alt. $\angle$s, $AB \parallel CD$)
   iii. $d + b = 180^\circ$ (int. $\angle$s, $AB \parallel CD$)
(e) $AB \parallel CD$ if
   i. $h = d$ (corr. $\angle$s equal)
   ii. $a = d$ (alt. $\angle$s equal)
   iii. $d + b = 180^\circ$ (int. $\angle$s equal)

Angles of a triangle and a convex polygon

(A) Triangle in general

(a) In $\triangle ABC$, $a + b + c = 180^\circ$. ($\angle$ sum of $\triangle$)
(b) If $BC$ is produced to $D$, then $d = a + b$ (ext. $\angle$ of $\triangle$)

(B) Isosceles Triangle

(a) Definition: An Isosceles Triangle is a triangle with 2 sides equal.
(b) If $AB = AC$, then $b = c$ (base $\angle$s, isos. $\triangle$)
(c) If $b = c$, then $AB = AC$ (sides opp. equal $\angle$s)
(C) Equilateral Triangle

(a) Definition: An equilateral triangle is a triangle with all 3 sides equal.

(b) If \( \Delta ABC \) is equilateral, the \( a = b = c = 60^\circ \) \textit{(property of equilateral \( \Delta \))}

(c) If \( a = b = c \), then \( \Delta ABC \) is equilateral.

(D) Convex polygon

(a) Definition: A convex polygon is a polygon in which each interior angle is less than \( 180^\circ \).

(b) The sum of all interior angles of a n-side polygon equals \( (n - 2) \times 180^\circ \). \textit{(\( \angle \) sum of polygon)}

(c) The sum of all exterior angles of a n-side polygon equals \( 360^\circ \) \textit{(sum of ext. \( \angle \)s of polygon)}

Similar Triangles and Congruent Triangles

(A) Similar Triangles

If \( \Delta ABC \sim \Delta XYZ \), then

(a) \( \angle A = \angle X \, , \, \angle B = \angle Y \, , \, \angle C = \angle Z \) \textit{(corr. \( \angle \)s, \( \sim \Delta \)s)}

(b) \( \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX} \) \textit{(corr. sides, \( \sim \Delta \)s)}

(B) Congruent Triangles

If \( \Delta ABC \cong \Delta XYZ \), then

(a) \( \angle A = \angle X \, , \, \angle B = \angle Y \, , \, \angle C = \angle Z \) \textit{(corr. \( \angle \)s, \( \cong \Delta \)s)}

(b) \( AB = XY, BC = YZ, CA = ZX \) \textit{(corr. sides, \( \cong \Delta \)s)}

Remarks:

(a) Name of Angles

i. \( 0^\circ < \text{acute angle} < 90^\circ \)

ii. \( 90^\circ < \text{obtuse angle} < 180^\circ \)

iii. \( 180^\circ < \text{reflex angle} < 360^\circ \)

iv. right angle = \( 90^\circ \)

v. straight angle = \( 180^\circ \)

(b) The sum of a pair of angles equal \( 90^\circ \), the pair of angles is called \textit{complementary} angles.

(c) The sum of a pair of angles equal \( 180^\circ \), the pair of angles is called \textit{supplementary} angles.

(d) There are totally five reasons to prove two triangles are congruent. They are \textit{SSS}, \textit{SAS}, \textit{ASA}, \textit{AAS}, \textit{RHS}.

(e) There are three reasons to show that two triangles are similar. They are \textit{AAA}, \textit{3 sides prop.}, \textit{ratio of 2 sides, inc. \( \angle \)}.

(f) All properties of parallelogram, rhombus, rectangle and square, please refer back to your textbook volume 3.

(g) And mid-point theorem, intercept theorem and equal ratio theorem, please also refer back to Book 3.